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Analytical Solution on the Wealth of Portfolios of investments for Capital Markets

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Abstract
To measure the wealth of a portfolio of investments refers to assessing the overall financial health and values of a collection of investments held by individual or organization; which allows investors to track the performance of their investments over time and understand the wealth on their investment. Therefore, this paper considered two system of second order Differential Equations (ODE) with appropriate stock quantities for variation of portfolios of investments. The problems were analytically solved by adopting the series solution approaches of Frobenius and method of undetermined coefficient; closed form solutions are derived for wealth of portfolios of investments. More so, capital market were effectively analyzed which demonstrated the impact analysis on the wealth of portfolios of investments and other capital market variables were presented graphically. Secondly, we state and prove theorem to show that our proposed model under-goes rate of change property which is informative to investors. To this end, the governing equations are reliable as it realistically addresses the principles of capital market investments.

Keywords: *Wealth, Investors, Stochastic Analysis, Capital Market and Portfolios*

1.1 Introduction

In real life generally, attempting to make profit are geared towards investing and trading respectively. Both investors and traders are the major participants in financial markets because is a means by which they earn a living through making profits. In all, investors deals with long term investment plans through buying and holding. While traders take advantage of rising and falling markets to enter and exit positions over a short period of time frame, taking smaller, more frequent profits. Thus, the benefit of investing is to gradually accommodate wealth over a long period of time through buying and holding of portfolios of investments. Still, making short or long term business plans needs financial assets which is propels bringing returns to the investors over time. Financial assets are the basis in which individual or group of persons can generate wealth to become comfortable in life.

However, mathematical analysis in finance is very important not only to mathematicians or investors but the generality of the masses. So understanding its dynamics will help economist, Government, opinion leaders to adequately plan their investments effectively well for future purposes. Hence, the special area of interest in this study is Mathematics of finance which has to deal with the evolution and phenomena of financial problems. More so, it is of great interest to also understand the dynamic nature of the effect of financial market. The ability to adequately understand financial variables, its dynamic relationships and how it affects investors or traders is of great practical importance. The mathematical modeling of financial quantities or variables has gained popularity due to its numerous applications in the fast growing field of science and technology. Such applications include the stock volatility which helps investors to measure raise and fall of stock prices, Insurance of risky investments, valuation of option prices for option traders etc. Now, in the case of asset value function, this needs some estimation in day to day activities in financial markets. Details of scholars who have dealt extensively in financial markets can be seen [1-9] and [11-17] etc.

This study is targeted to consider two system of 2nd order ordinary differential equations to quantify each portfolios of investment via stock variables by means of closed form analytical solutions. It has been known that investors are really disadvantaged in their primary decisions due to expected returns. This encouraged the authors of this paper to come up with a vital approach that will help in decision making.

It is obvious that [10] has considered stochastic model of market assessments of stock returns and value of asset prices in time-varying investment returns which follows multiplicative, additive effects series with quadratic functions. The advantage of current paper over [10] the return effects were removed and replaced with interest rates to assess the wealth function of corporate investors. Our novel idea compliments previous discovery in this dynamic area of mathematical finance.

To this end, the paper entails to be arranged as follows: 2.1 presents materials and methods, the results and discussion is seen in 3.1, the paper is concluded in 4.1.

2.1 Materials and Methods

Here, we state few definitions which captures the foundations of this paper.

Definition 1.1. Stochastic process: A relation of random variables $\{X_t,(r),t\in T$ is a stochastic process $X(t)$. So the variable have t to represents time and $X(t)$ as the state of the process at time t.

Definition 1.2 .Stochastic Differential Equation (SDE): Now, let $S(t)$ denotes the price of risky asset at particular time t, and μ, an expected rate of returns on the stock and dt is the significance change during the period of trading such that the stock becomes random walk which is systematically given as SDE.

$$dS(t)=\mu S(t)dt+\sigma S(t)dW_t$$

Where, μ represents drift and σ is the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega,\xi,\wp),\xi$ is a $\sigma-$algebra generated by $W_t, t\geq 0$.The details of above definitions are seen as follows [5-7] etc.

2.1.1 Problem Formulation of Investment Equations

Here, considering companies with disparities of portfolios in an investments such as V_1 and V_2 be represented in the systems of 2nd order differential equation. However, the above concepts is quantitative since prices observes by investors prices and takes action in discrete time periods $t=0,1,2,3,4,...,T$, the price changes and fundamental factors are very unclear and are displayed in probability space, [16]. The dynamics governing this processes are as follows:

$$S\frac{d^2V_1}{dS^2}+\frac{dV_1}{dS}-\sigma^2V_1=-\rho-IrV_2(t) \tag{1}$$

$$S\ \frac{d^2V_2}{dS^2}+\frac{dV_2}{dS}-\lambda^2V_2=0 \tag{2}$$

With the following boundary conditions

$$\left.\begin{array}{l} V_1=0,\ V_2=V_{2\alpha},,V_4,S=1 \\ \dfrac{dV_1}{dS}=\dfrac{dV_2}{dS}=0 \end{array}\right\} \tag{3}$$

Where V_1, and V_2 represents the underlying assets p and Ir_t is constant parameters, λ is the interest rate parameter of investments, σ .is the volatility ,and S is the price proces of the stock market.

2.1.2 Method of Solution

The proposed model (1)-(2) consist of investment equations which is not too difficult to solve independently. Adopting the Frobenius method for solving $V_1(S)$,and $V_2(S)$ Firstly, we solve all the homogenous part of the equations. In addressing these challenges effectively, it's important to recognize that

$\mathrm{V}_1(\mathrm{S}),\ \ \mathrm{V}_2(\mathrm{S}),\ \ <\infty$ for all $S\in[0,1)$

2.1.3 The Portfolios of Investment for First Corporate Investors:

From the homogenous part of (1)

$$\text{Let}\ \ V_1=\sum_{m=0}^{\infty}a_mS^{m+c}=S^c\sum_{m=0}^{\infty}a_mS^m \tag{2}$$

$$V_1'=\sum_{m=0}^{\infty}a_m(m+c)S^{m+c-1}=S^{c=1}\sum_{n=0}^{\infty}a_m(m+c)S^m \tag{3}$$

$$V_1''=S_1^{c-2}\sum_{m=0}^{\infty}a_m(m+c)(m+c-1)S^m=S^{c-1}\sum_{m=0}^{\infty}a_m(m+c)(m+c-1)S^m \tag{4}$$

substituting (4)-(7) into (1) and performing some algebraic operations gives

$$V_1(t)=S^c\left\{\begin{array}{l} a_0+\dfrac{\sigma^2a_0S^2}{(c+2)^2}+\dfrac{\sigma^4a_0S^4}{(c+2)^2(c+4)^2}+\dfrac{\sigma^6a_0S^6}{(c+2)^2(c+4)^2(c+6)^2}+\dfrac{\sigma^8a_0S^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2} \\ +\dfrac{\sigma^{10}a_0S^{10}}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2(c+10)^2}+\cdots \end{array}\right\}$$

$$V_{1(t)}(S)=v_1=\alpha_1\left\{\begin{array}{l} 1+\dfrac{\sigma^2S^2}{2^2}+\dfrac{\sigma^4S^4}{2^2\times4^2}+\dfrac{\sigma^6S^6}{2^2\times4^2\times6^2}+\dfrac{\sigma^8S^8}{2^2\times4^2\times6^2\times8^2} \\ +\dfrac{\sigma^{10}S^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}+\ldots \end{array}\right\} \tag{5}$$

Another is given by $v_2=\dfrac{dV_{1(t)}}{dc}$

$$\frac{dV_{1(t)}}{dc} = a_0 S^c lnr \left\{ 1 + \frac{\sigma^2 S^2}{(c+2)^2} + \frac{\sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{\sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right.$$

$$\left. + \frac{\sigma^8 S^8}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{\sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} .. \right\}$$

$$+ a_0 S^c \frac{d}{dc} \left\{ 1 + \frac{\sigma^2 S^2}{(c+2)^2} + \frac{\sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{\sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right.$$

$$\left. + \frac{\sigma^8 S^8}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{\sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} + \dots \right\}$$

$$\frac{dV_{1(t)}}{dc} = a_0 S^c \ln S \left\{ 1 + \frac{\sigma^2 S^2}{(c+2)^2} + \frac{\sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{\sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right.$$

$$\left. + \frac{\sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{\sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} + \dots \right\}$$

$$+ a_0 r^c \left\{ 0 - \frac{2\sigma^2 S^2}{(c+2)^3} - \frac{4\sigma^4 S^4 (c+3)}{(c+2)^3 (c+4)^3} - \frac{8\sigma^6 S^6}{(c+2)^3 (c+4)^3 (c+6)^3} + \frac{16\sigma^8 S^8}{(c+2)^3 (c+4)^3 (c+6)^3 (c+8)^3} \right.$$

$$\left. - \frac{32\sigma^{10} S^{10}}{(c+2)^3 (c+4)^3 (c+6)^3 (c+8)^3} + \dots \right\}$$

When $c = 0$

$$V_1 = v_2 = \alpha_2 \left\{ \begin{array}{l} \ln S \left(\begin{array}{l} 1 + \frac{\sigma^2 S^2}{2^2} + \frac{\sigma^4 S^4}{2^2 \times 4^2} + \frac{\sigma^6 S^6}{2^2 \times 4^2 \times 6^2} + \frac{\sigma^8 S^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{\sigma^{10} S^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{array} \right) \\ + a_0 S^c \left(\begin{array}{l} - \frac{\sigma^2 S^2}{2^2} - \frac{\sigma^4 S^4}{2^3 \times 4^2} - \frac{\sigma^6 S^6}{4^3 \times 6^3} + \frac{\sigma^8 S^8}{2^5 \times 6^3 \times 8^3} \\ - \frac{\sigma^{10} S^{10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \dots \end{array} \right) \end{array} \right\} \quad (6)$$

A linear combination (5) and (6) gives the complete solution

$$V_1(S)=\alpha_1\left\{\begin{array}{l}1+\dfrac{\sigma^2S^2}{2^2}+\dfrac{\sigma^4S^4}{2^2\times4^2}+\dfrac{\sigma^6S^6}{2^2\times4^2\times6^2}+\dfrac{\sigma^8S^8}{2^2\times4^2\times6^2\times8^2}\\+\dfrac{\sigma^{10}S^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}+\ldots\end{array}\right\}$$

$$+\alpha_2\left\{\begin{array}{l}\ln S\left(\begin{array}{l}1+\dfrac{\sigma^2S^2}{2^2}+\dfrac{\sigma^4S^4}{2^2\times4^2}+\dfrac{\sigma^6S^6}{2^2\times4^2\times6^2}+\dfrac{\sigma^8S^8}{2^2\times4^2\times6^2\times8^2}\\+\dfrac{\sigma^{10}S^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}\end{array}\right)\\-\dfrac{\sigma^2S^2}{2^2}-\dfrac{\sigma^4S^4}{2^3\times4^2}-\dfrac{\sigma^6S^6}{4^3\times6^3}+\dfrac{\sigma^8S^8}{2^5\times6^3\times8^3}-\dfrac{\sigma^{10}S^{10}}{4^2\times6^3\times8^3\times10^3}+\ldots\end{array}\right\} \quad (7)$$

Applying the boundary conditions in (3) and setting $\alpha_2=0$ gives

$$V_1(S)=\left\{1+\frac{\sigma^2S_1^{\,2}}{2^2}+\frac{\sigma^4S^4}{2^2\times4^2}+\frac{\sigma^6S^6}{2^2\times4^2\times6^2}+\frac{\sigma^8S^8}{2^2\times4^2\times6^2\times8^2}+\frac{\sigma^{10}S^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}+\ldots\right\} \quad (8)$$

2.1.4 :The Portfolios of Investment for Second Corporate Investors:

From the homogenous part of (2)

Let $$V_2=\sum_{m=0}^{\infty}a_mS^{m+c}=S^c\sum_{m=0}^{\infty}a_mS^m \quad (9)$$

$$V_2'=\sum_{m=0}^{\infty}a_m\left(m+c\right)S^{m+c-1}=S^{c=1}\sum_{n=0}^{\infty}a_m\left(m+c\right)S^m \quad (10)$$

$$V_2''=S_1^{c-2}\sum_{m=0}^{\infty}a_m\left(m+c\right)\left(m+c-1\right)S^m=S^{c-1}\sum_{m=0}^{\infty}a_m\left(m+c\right)\left(m+c-1\right)S^m \quad (11)$$

substituting (9)-(11) into (2) and performing some algebraic operations gives

$$S^{c-1}\sum_{m=0}^{\infty}a_m\left(m+c\right)\left(m+c-1\right)S^m+S^{c-1}\sum_{m=0}^{\infty}a_m\left(m+c\right)S^m-\sum_{m=0}^{\infty}\lambda^2a_mS^{m+c+1}=0 \quad (12)$$

$$S^{c-1}\sum_{m=0}^{\infty}a_m\left(m+c\right)^2S^m-\sum_{m=0}^{\infty}\lambda^2a_mS^{m+c+1}=0 \quad (13)$$

Setting $k=n-1\Rightarrow n=k+1$

$$S^{c-1}a_0c^2+\sum_{k=0}^{\infty}a_{k+1}\left(k+c+1\right)^2S^{k+c}-\sum_{n=0}^{\infty}a_n\lambda^2S^{n+c+1}=0$$

$$a_0c^2S^{c-1}+\sum_{n=0}^{\infty}\left(a_{n+1}\left(n+c+1\right)^2S^{n+c}-a_n\lambda^2S^{n+c+1}\right)=0$$

$$V_1 = v_2 = \alpha_2 \left\{ \begin{array}{l} \ln S \left(\begin{array}{l} 1 + \frac{\lambda^2 S^2}{2^2} + \frac{\lambda^4 S^4}{2^2 \times 4^2} + \frac{\lambda^6 S^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8 S^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{\lambda^{10} S^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{array} \right) \\ + a_0 S^c \left(\begin{array}{l} - \frac{\lambda^2 S^2}{2^2} - \frac{\lambda^4 S^4}{2^3 \times 4^2} - \frac{\lambda^6 S^6}{4^3 \times 6^3} + \frac{\lambda^8 S^8}{2^5 \times 6^3 \times 8^3} \\ - \frac{\lambda^{10} S^{10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \ldots \end{array} \right) \end{array} \right\} \quad (21)$$

A linear combination (20) and (21) gives the complete solution

$$V_2(S) = \alpha_1 \left\{ \begin{array}{l} 1 + \frac{\lambda^2 S^2}{2^2} + \frac{\lambda^4 S^4}{2^2 \times 4^2} + \frac{\lambda^6 S^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8 S^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{\lambda^{10} S^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{array} \right\}$$

$$+ \alpha_2 \left\{ \begin{array}{l} \ln S \left(\begin{array}{l} 1 + \frac{\lambda^2 S^2}{2^2} + \frac{\lambda^4 S^4}{2^2 \times 4^2} + \frac{\lambda^6 S^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8 S^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{\lambda^{10} S^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{array} \right) \\ - \frac{\lambda^2 S^2}{2^2} - \frac{\lambda^4 S^4}{2^3 \times 4^2} - \frac{\lambda^6 S^6}{4^3 \times 6^3} + \frac{\lambda^8 S^8}{2^5 \times 6^3 \times 8^3} - \frac{\lambda^{10} S^{10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \ldots \end{array} \right\} \quad (17)$$

Applying the boundary conditions in (3) and setting $\alpha_2 = 0$ gives

$$\alpha_1 = \frac{V_{2\alpha}}{y_1(1)} \quad (18)$$

$$where \;\; y_{(1)} = \left\{ 1 + \frac{\lambda^2}{2^2} + \frac{\lambda^4}{2^2 \times 4^2} + \frac{\lambda^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{\lambda^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \right\}$$

$$V_2(S) = \frac{V_{2\alpha}}{y_1(1)} \left\{ \begin{array}{l} 1 + \frac{\lambda^2 S^2}{2^2} + \frac{\lambda^4 S^4}{2^2 \times 4^2} + \frac{\lambda^6 S^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8 S^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{\lambda^{10} S^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{array} \right\} (19)$$

2.1.5 Non-homogenous part of investment equation for The Portfolios of Investment for First Corporate Investors:

From RHS of (1) using the method of undetermined coefficient

$$V_2(S) = \alpha_1 \left\{ 1 + \frac{\lambda^2 S^2}{2^2} + \frac{\lambda^4 S^4}{2^2 \times 4^2} + \frac{\lambda^6 S^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8 S^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{\lambda^{10} S^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \right\}$$

The complementary function for (8)

$$V_{1c}\left(S\right)=C_c\left[1+\frac{\sigma^2S^2}{2^2}+\frac{\sigma^4S^4}{\left(2\times4\right)^2}+\frac{\sigma^6S^6}{\left(2\times4\times6\right)^2}+\frac{\sigma^8S^8}{\left(2\times4\times6\times8\right)^2}+\frac{\sigma^{10}S^{10}}{\left(2\times4\times6\times8\times10\right)^2}+\ldots\right] \quad (20)$$

Consider the particular solution

$$V_p\left(S\right)=A_0+A_1S^2+A_2S^4+A_3S^6+A_4S^8+A_5S^{10} \quad (21)$$

$$V_p'\left(S\right)=2A_1S+4A_2S^3+6A_3S^5+8A_4S^7+10A_5S^9 \quad (22)$$

$$V_p''\left(S\right)=2A_1+12A_2S^2+30A_3S^4+56A_4S^6+90A_5S^8 \quad (23)$$

Putting (21)-(23) into (1) gives

$$\Rightarrow 4A_1+16A_2S^2+36A_3S^4+64A_4S^6+100A_5S^8-\sigma^2A_0-\sigma^2A_1S^2$$
$$-\sigma^2A_2S^4-\sigma^2A_3S^6-\sigma^2A_4S^8-\sigma^2A_5S^{10}\equiv$$
$$-\varphi-Ir_tV_2\left(1+\frac{\{\lambda S\}^2}{2^2}+\frac{\{\lambda S\}^4}{(2\times4)^2}+\frac{\{(\lambda S\}^6}{(2\times4\times6)^2}+\frac{\{(\lambda S\}^8}{(2\times4\times6\times8)^2}+\frac{\{(\lambda S\}^{10}}{(2\times4\times6\times8\times10)^2}\right)$$

(24)

$$RHS=\left(4A_1-(\sigma)^2A_0\right)+\left(16A_2-(\sigma)^2A_1\right)S^2+\left(36A_3-(\sigma)^2A_2\right)S^4$$
$$+\left(64A_4-(\sigma)^2A_3\right)S^6+(100\mathrm{A}_5-(\sigma)^2A_4)\mathrm{S}^8-(\sigma)^2A_5S^{10} \quad (25)$$

Combining (224) and (25) yields the following results:

$$4A_1-\sigma^2A_0=-\varphi-Ir_tV_2 \quad (26)$$

$$16A_2-\{\sigma\}^2A_1=-\frac{\lambda^2Ir_tV_2}{4} \quad (27)$$

$$36A_3-\{\sigma\}^2A_2=-\frac{\lambda^4Ir_tV_2}{\left(2\times4\right)^2} \quad (28)$$

$$64A_4-\{\sigma\}^2A_3=-\frac{\lambda^6Ir_tV_2}{\left(2\times4\times6\right)^2} \quad (29)$$

$$100A_5-\{\sigma\}^2A_4=-\frac{\lambda^8Ir_tV_2}{\left(2\times4\times6\times8\right)^2} \quad (30)$$

$$-\{\sigma\}^2A_5=-\frac{\lambda^{10}Ir_tV_2}{\left(2\times4\times6\times8\times10\right)^2} \quad (31)$$

To get the constant c , we combine(30) and (31) to gives:

$$V(S) = c\left[\begin{array}{l} 1 + \frac{(\sigma)^2 S^2}{4} + \frac{(\sigma)^4 S^4}{(2\times4)^2} + \frac{(\sigma)^6 S^6}{(2\times4\times6)^2} + \frac{(\sigma)^8 S^8}{(2\times4\times6\times8)^2} \\ + \frac{(\sigma)^{10} S^{10}}{(2\times4\times6\times8\times10)^2} \end{array}\right] + A_0 + A_1 S^2 + A_2 S^4 + A_3 S^6 + A_4 S^8 + A_4 S^{10} \tag{32}$$

Applying boundary condition (3) on (32) gives

$$0 = V(a) = c\left[\begin{array}{l} 1 + \frac{(\sigma)^2 a^2}{4} + \frac{(\sigma)^4 a^4}{(2\times4)^2} + \frac{(\sigma)^6 a^6}{(2\times4\times6)^2} + \frac{(\sigma)^8 a^8}{(2\times4\times6\times8)^2} \\ + \frac{(\sigma)^{10} a^{10}}{(2\times4\times6\times8)^2} + \ldots \end{array}\right] + A_0 + A_1 a^2 + A_2 a^4 + A_3 a^6 + A_4 a^8 + A_5 a^{10}$$

$$= c\left[\frac{A_0 + A_1 a^2 + A_2 a^4 + A_3 a^6 + A_4 a^8 + A_5 a^{10}}{\begin{array}{l} 1 + \frac{(\sigma)^2 a^2}{4} + \frac{(\sigma)^4 a^4}{(2\times4)^2} + \frac{(\sigma)^6 a^6}{(2\times4\times6)^2} + \frac{(\sigma)^8 a^8}{(2\times4\times6\times8)^2} \\ + \frac{(\sigma)^{10} a^{10}}{(2\times4\times6\times8)^2} \end{array}}\right] \tag{33}$$

From (33)

$$V_1(S) = \left[\begin{array}{l} c + \frac{c(\sigma)^2 S^2}{4} + \frac{c(\sigma)^4 S^4}{(2\times4)^2} + \frac{c(\sigma)^6 S^6}{(2\times4\times6)^2} + \frac{c(\sigma)^8 S^8}{(2\times4\times6\times8)^2} \\ + \frac{c(\sigma)^{10} S^{10}}{(2\times4\times6\times8\times10)^2} \end{array}\right] + A_0 + A_1 S_3{}^2 + A_2 S_3{}^4 + A_3 S_3{}^6 + A_4 S_3{}^8 + A_5 S_3{}^{10}$$

Taking like terms gives a complete solution

$$V_1(S) = c + A_0 + \left(\frac{c(\sigma)^2}{4} + A_1\right)S^2 + \left(\frac{c(\sigma)^4}{(2\times4)^2} + A_2\right)S^4 + \left(\frac{c(\sigma)^6}{(2\times4\times6)^2} + A_3\right)S^6 + \left(\frac{c(\sigma)^8}{(2\times4\times6\times8)^2} + A_4\right)S^8 + \left(\frac{c(\sigma)^{10}}{(2\times4\times6\times8\times10)^2} + A_5\right)S^{10} \tag{34}$$

where $A_0, A_1, A_2, A_3, A_4,$ and A_5 are constants which is seen in appendix

2.2 Analysis of the Analytical Solutions

Here, the solutions of the four stock variables will be subjected for analysis in other to ascertain its future rate of change property in respect to asset prices. Hence, we state theorem as follows:
Theorem 1.1 (Rate of Change Property): The analytical solution of equation (19) and (34) derived for each of portfolios of investments. That is:

$$(i)\ V_1(S) = c + A_0 + \left(\frac{c(\sigma)^2}{4} + A_1\right)S^2 + \left(\frac{c(\sigma)^4}{(2\times4)^2} + A_2\right)S^4 + \left(\frac{c(\sigma)^6}{(2\times4\times6)^2} + A_3\right)S^6$$
$$+\left(\frac{c(\sigma)^8}{(2\times4\times6\times8)^2} + A_4\right)S^8 + \left(\frac{c(\sigma)^{10}}{(2\times4\times6\times8\times10)^2} + A_5\right)S^{10}$$

$$\text{(ii) } V_2(S) = \frac{V_{2\alpha}}{y_1(1)}\left\{1 + \frac{\lambda^2 S_1^2}{2^2} + \frac{\lambda^4 S^4}{2^2\times4^2} + \frac{\lambda^6 S^6}{2^2\times4^2\times6^2} + \frac{\lambda^8 S^8}{2^2\times4^2\times6^2\times8^2} + \frac{\lambda^{10} S^{10}}{2^2\times4^2\times6^2\times8^2\times10^2} + \ldots\right\}$$

Proof; : We want to show different asset price rate of change for independent portfolios

$$V_1'(S) = 2\left(\frac{c(\sigma)^2}{4} + A_1\right)S + 4\left(\frac{c(\sigma)^4}{(2\times4)^2} + A_2\right)S^3 + 6\left(\frac{c(\sigma)^6}{(2\times4\times6)^2} + A_3\right)S^5 + 8\left(\frac{c(\sigma)^8}{(2\times4\times6\times8)^2} + A_4\right)S^7$$
$$+10\left(\frac{c(\sigma)^{10}}{(2\times4\times6\times8\times10)^2} + A_5\right)S^9$$

$$V_1''(S) = 2\left(\frac{c(\sigma)^2}{4} + A_1\right) + 12\left(\frac{c(\sigma)^4}{(2\times4)^2} + A_2\right)S^2 + 30\left(\frac{c(\sigma)^6}{(2\times4\times6)^2} + A_3\right)S^4 + 56\left(\frac{c(\sigma)^8}{(2\times4\times6\times8)^2} + A_4\right)S^6$$
$$+90\left(\frac{c(\sigma)^{10}}{(2\times4\times6\times8\times10)^2} + A_5\right)S^8$$

Similarly (ii)

$$V_2'(S) = \frac{2\lambda S}{4} + \frac{4(\lambda S)^3}{2^2\times4^2} + \frac{6(\lambda S)^5}{2^2\times4^2\times6^2} + \frac{8(\lambda S)^7}{2^2\times4^2\times6^2\times8^2} + \frac{10(\lambda S)^9}{2^2\times4^2\times6^2\times8^2\times10^2}$$
$$V_2''(S) = \frac{2\lambda}{4} + \frac{12(\lambda S)^2}{2^2\times4^2} + \frac{30(\lambda S)^4}{2^2\times4^2\times6^2} + \frac{56(\lambda S)^6}{2^2\times4^2\times6^2\times8^2} + \frac{90(\lambda S)^9}{2^2\times4^2\times6^2\times8^2\times10^2}$$

Therefore, the claim is true.

3.1 Results and Discussion

We present the computational results for the problem formulated in (1)-(2) as follows.
The solutions are implemented in a python programming language and the graphical results are obtained using Matplotlib. However, the following parameter values were explicitly used in the simulation study:

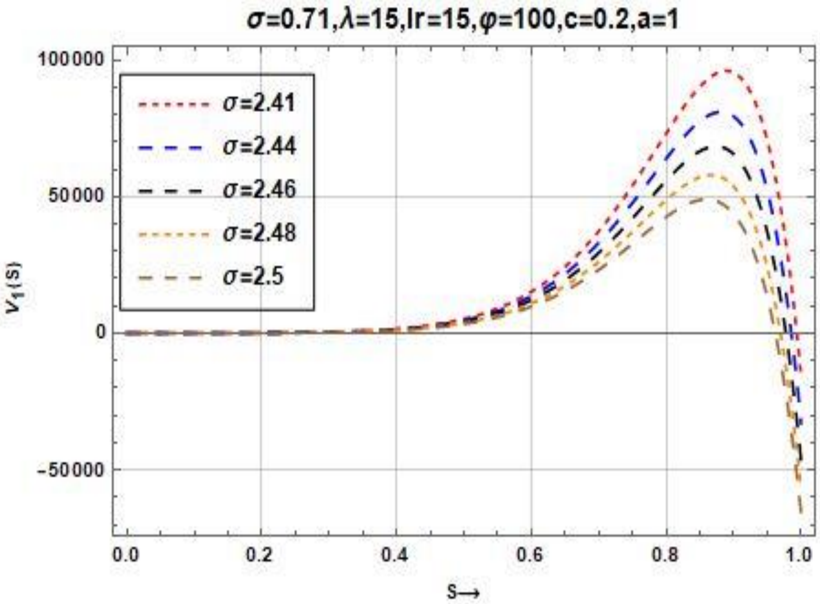

Figure 1: Wealth of corporate investor profiles against S with variations of volatility parameter

Figure 1 shows increase in volatility decreases the wealth of corporate investor. Generally, that statement is true . Increase volatility in the market tends to make investing more risks, which can lead to decreased wealth for investors. High volatility makes it more difficult to predict future outcomes and can lead to sudden and dramatic changes in the value of assets, which can negatively impact an investor's portfolio. Investors who prefer low-risk investments may be especially vulnerable to losses in volatile markets, as they may be unable to take advantage of rapid price changes or may be forced to sell assets at a loss to avoid further losses. Overall, increase volatility is generally seen as a negative factor for most investors.

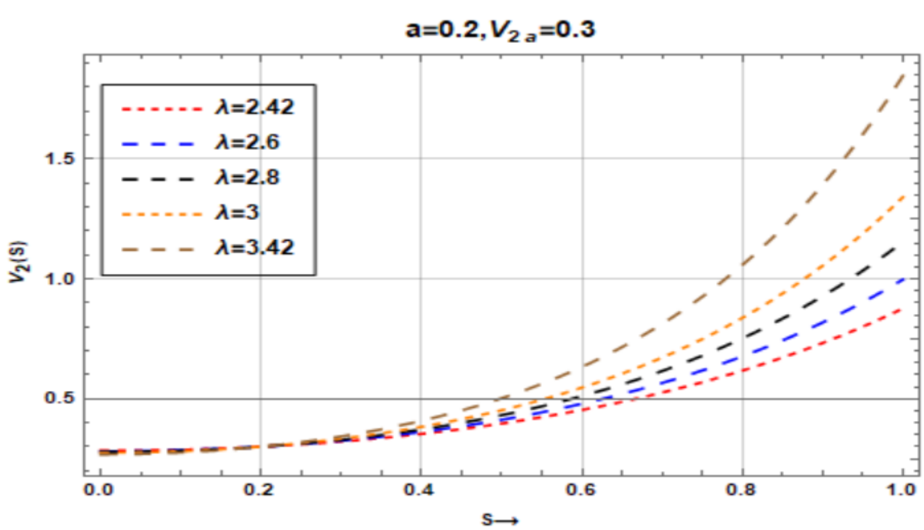

Figure 2: Wealth of corporate investor profiles against S with variations of interest rate parameter

Figure 2 shows increase in the interest rates also increases the wealth of investors. Higher interest rates can provide better returns on investments such as bonds and savings accounts, they can also negatively impact other types of investments. For example, higher interest rates can lead to lower stock prices, as companies may have to pay more to borrow money, which can reduce their profitability. Additionally, higher interest rates can lead to a stronger currency, which can negatively impact exports and hurt companies that rely on oversees sales. Overall, the impact of higher interest rates on investor wealth depend on the specific investments and economic conditions at the time.

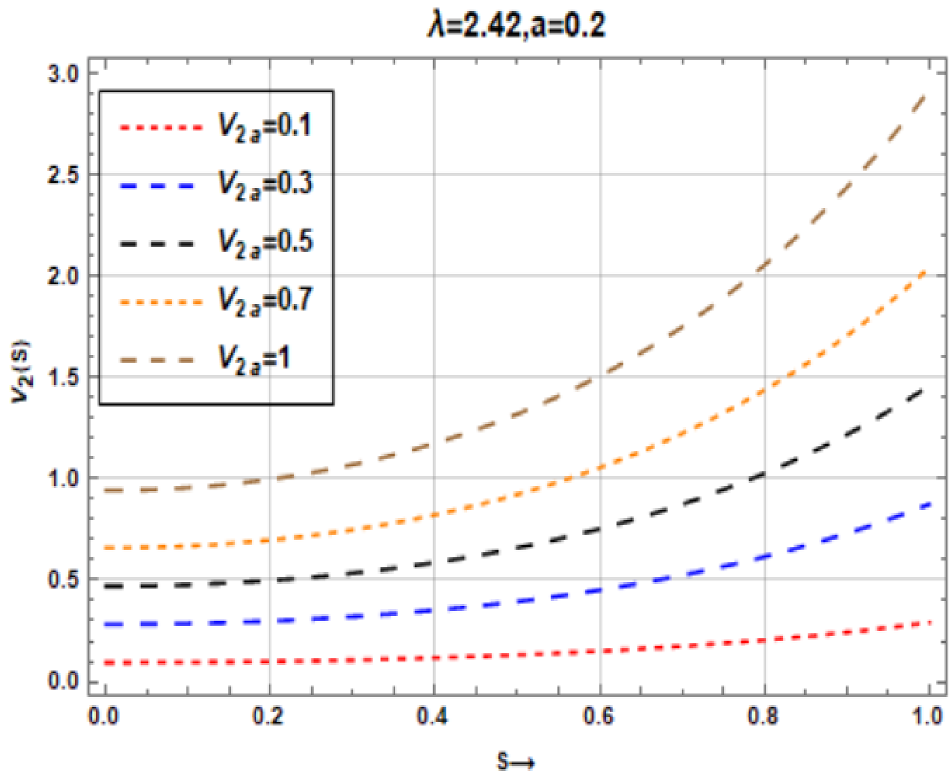

Figure 3: Wealth of corporate investor profiles against S with variations of boundary condition parameter

In Figure 3, shows increasing in the boundary condition increases the wealth of investments (perhaps by putting more money into stocks, bonds, or other investments) can increase the wealth of investors. In general, this can be true, as increasing the amount of money invested can allow for greater potential returns. However, it's important to keep in mind that investing carries inherent risk, and there is no gurantee that increasing the amount of investment will always lead to greater wealth. Its important to consider your personal financial goals and risk tolerance when making investment decisions

4.1 Conclusion

In this paper, we studied the wealth of corporate investments for capital market prices. The two differential equations is proposed for the problem and analytical solution is derived in details which shows the following results: (i) increase in volatility decreases the wealth of corporate investor. (ii) increase in the interest rates also increases the wealth of investors (iii) increasing in the boundary condition increases the wealth of corporate investors (iv) the governing equations are reliable as it realistically addresses the principles of capital market investments. Using two dimensional cases of drifts in assessing wealth of corporate investors is highly recommended.

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Appendix

$$\Rightarrow A_5 = -\frac{1}{(\sigma)^2}\left[\frac{\lambda^{10} Ir_t V_2}{(2\times4\times6\times8\times10)^2}\right], \Rightarrow A_4 = -\frac{1}{(\sigma)^2}\left[100A_5 - \frac{\lambda^8 Ir_t V_2}{(2\times4\times6\times8)^2}\right]$$

$$\Rightarrow A_3 = -\frac{1}{(\sigma)^2}\left[64A_4 - \frac{\lambda^6 Ir_t V_2}{(2\times4\times6)^2}\right], \Rightarrow A_2 = -\frac{1}{(\sigma)^2}\left[36A_3 - \frac{\lambda^4 Ir_t V_2}{(2\times4)^2}\right]$$

$$\Rightarrow A_1 = -\frac{1}{(\sigma)^2}\left[16A_2 - \frac{\lambda^2 Ir_t V_2}{4}\right], \Rightarrow A_0 = -\frac{1}{(\sigma)^2}\left[4A_1 + \varphi - Ir_t V_2\right]$$